

Strategic abstention based on preference extensions: Positive results and computer-generated impossibilities

FLORIAN BRANDL, FELIX BRANDT, CHRISTIAN GEIST, JOHANNES HOFBAUER

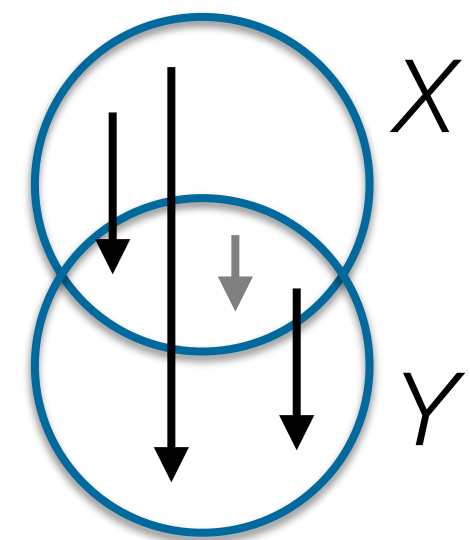
Preliminaries

- A **social choice function (SCF)** maps every preference profile to a subset of the alternatives.
- An SCF is **majoritarian** if it only depends on the (unweighted) majority comparisons between alternatives.
- An SCF satisfies **independence of indifference voters (IIV)** if an agent who is indifferent among all alternatives does not change the outcome.
- A preference profile R' is an **f -improvement** over R if alternatives in $f(R)$ are not weakened from R to R' . An SCF f satisfies **set-monotonicity** if $f(R) = f(R')$ whenever R' is an f -improvement over R .
- An SCF is **Pareto-optimal** if it never selects a Pareto-dominated alternative.

Preference Extensions

A **preference extension** extends preferences over alternatives to (possibly incomplete) preferences over sets of alternatives.

- **Kelly's extension:** $X \succeq_i^K Y$ iff $x \succeq_i y$ for all $x \in X$ and $y \in Y$.
- **Fishburn's Extension:** $X \succeq_i^F Y$ iff $X \setminus Y \succeq_i^K Y$ and $X \succeq_i^K Y \setminus X$.



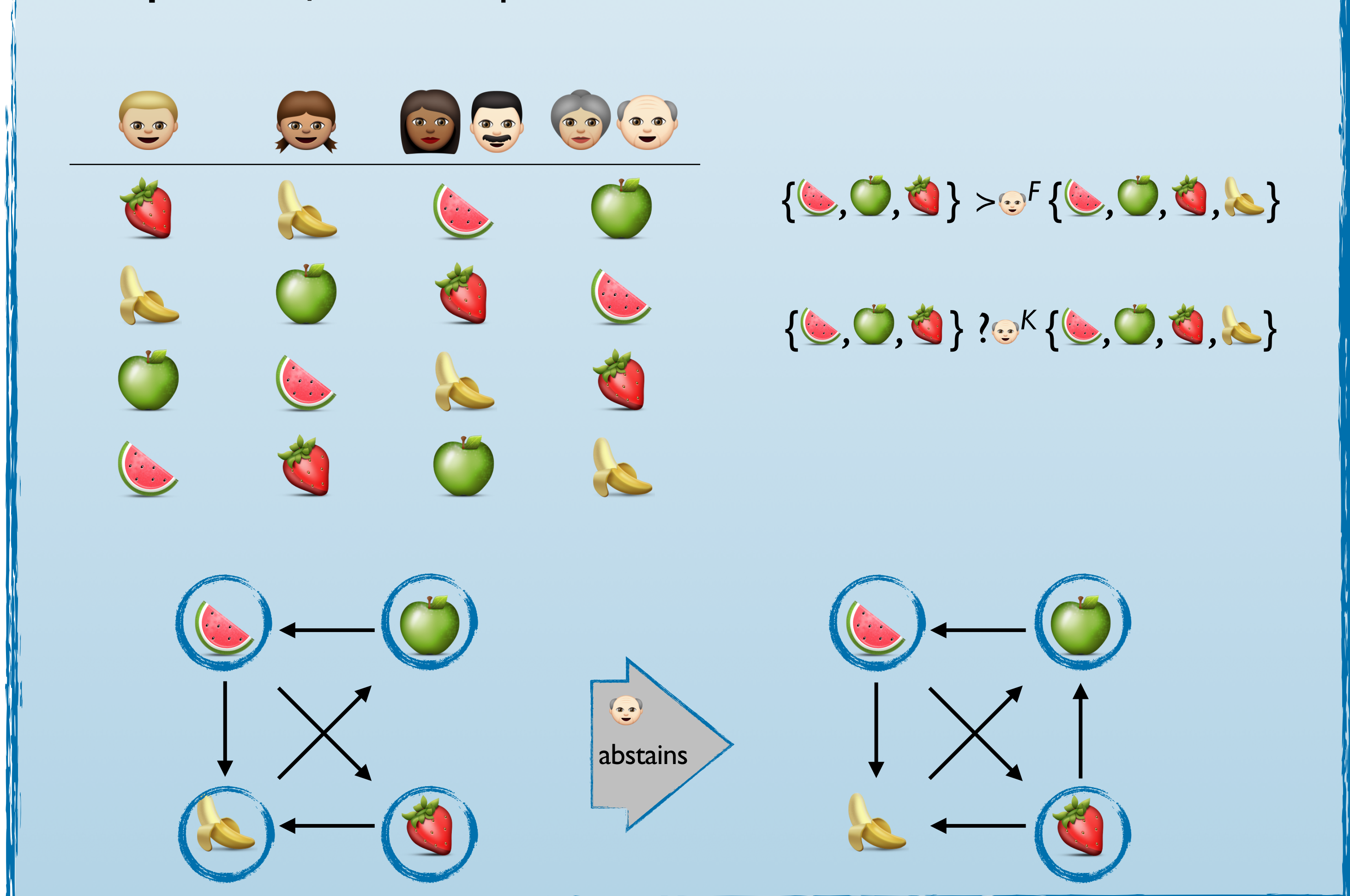
Participation

Participation prescribes that no agent can obtain a better outcome by abstaining from an election. Formally, an SCF f satisfies

- **Kelly-participation** if there is no preference profile R such that $f(R_{-i}) \succeq_i^K f(R)$.
- **Fishburn-participation** if there is no preference profile R such that $f(R_{-i}) \succeq_i^F f(R)$.

Fishburn-participation implies Kelly-participation.

Example: Let f be the bipartisan set.



Results

- There is no majoritarian and Pareto-optimal SCF that satisfies Fishburn-participation (for four or more alternatives).
- There is no majoritarian and Pareto-optimal SCF that satisfies Fishburn-participation even if preferences are strict (for five or more alternatives).
- Every SCF that satisfies IIV and set-monotonicity satisfies Kelly-participation.

Computer-aided Theorem Proving

This technique yields **full results** plus the possibility to extract **human-readable proofs** (from a minimal unsatisfiable set [MUS]).



Check of finite instance using a SAT solver

- **Fixed number of alternatives m**
 - Still very large (e.g., 5 alternatives \rightarrow about 10^{868} majoritarian SCFs)
 - Voters abstracted away by equivalent property: **majority-participation**
- Method: **Encode framework and properties into SAT**
 - Functionality, Neutrality
 - Participation, Pareto optimality

SAT

SCF with the required properties

UNSAT

Inductive step

- m alternatives $\Rightarrow m+1$ alternatives
- Manual (but simple) proof

Summary & Outlook

		Strict preferences	Weak preferences
Kelly	Participation	+	+
	Strategy-proofness	+	-
Fishburn	Participation	-	-
	Strategy-proofness	-	-

Additional context from image: Kelly Participation is linked to AAMAS-14 and IJCAI-11. Fishburn Participation is linked to AAMAS-14. Kelly Participation is linked to IJCAI-11. Fishburn Participation is linked to AAMAS-14. Kelly Participation is linked to IJCAI-11. Fishburn Participation is linked to AAMAS-14.

Two questions for future research:

- Do the impossibilities still hold for pairwise functions?
- Can we get positive results with even weaker requirements?