

# Incentives for Participation and Abstention in Probabilistic Social Choice

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## Preliminaries

- A **social decision scheme (SDS)** is a function that maps a preference profile to a lottery over the alternatives. Formally, an SDS is a function  $f: \mathcal{R}^{\mathcal{F}(\mathcal{N})} \rightarrow \Delta(A)$ .
- An SDS is **majoritarian** if it only depends on the (unweighted) majority comparisons between alternatives.
- We compare lotteries using **stochastic dominance (SD)**. A lottery  $p$  is preferred to another lottery  $q$  if the expected utility for  $p$  is at least as large as that for  $q$  for every von Neumann-Morgenstern utility function. Formally,
 
$$p \succeq q \text{ iff } \sum_{x \succeq y} p(x) \geq \sum_{x \succeq y} q(x) \text{ for all } y \in A$$
- **Random serial dictatorship (RSD)** is the generalization of random dictatorship to weak preferences. RSD is defined by picking a sequence of voters uniformly at random and then invoking serial dictatorship.
- **BOR** returns the uniform lottery over all Borda winners, i.e., alternatives that receive the highest Borda score.

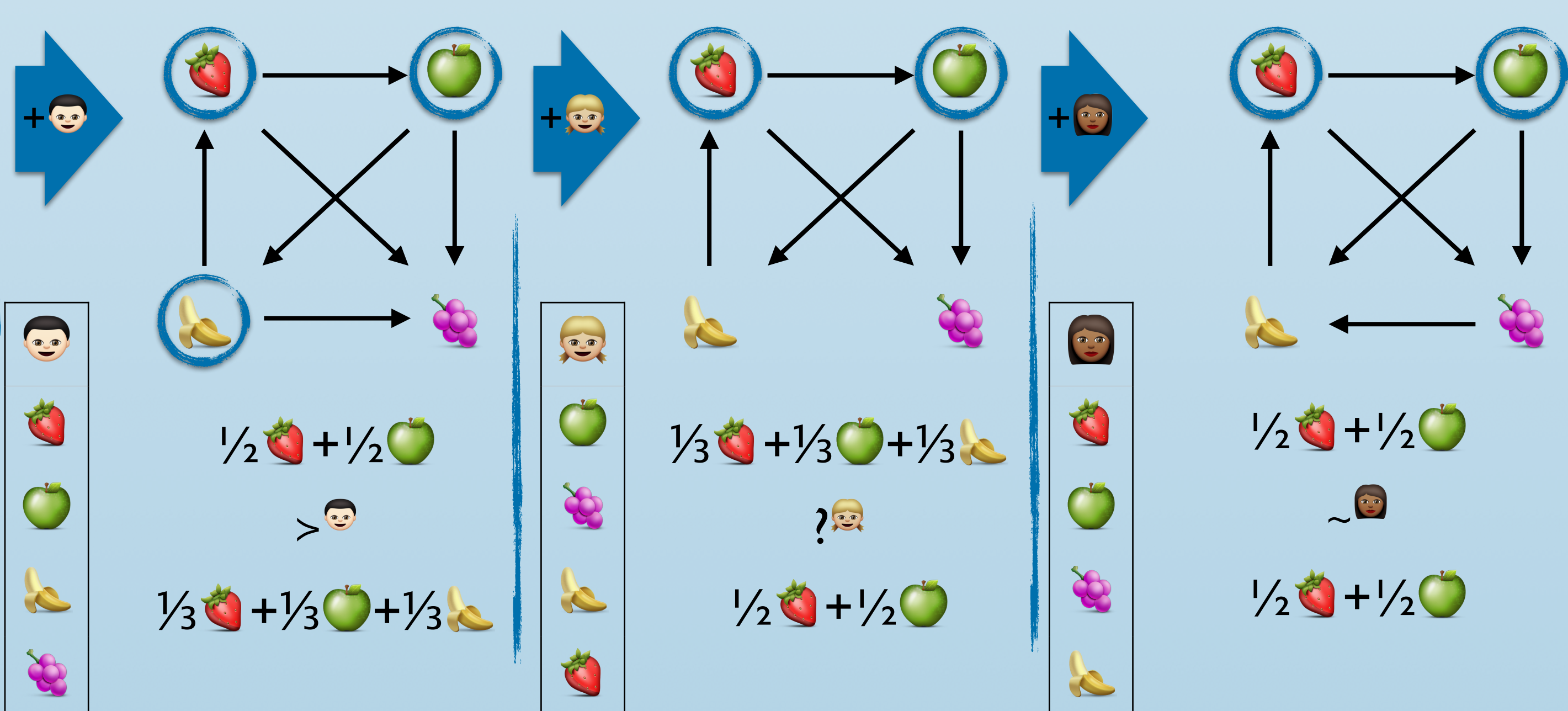
## Degrees of Participation

- **Participation:** Voting is **never worse** than abstaining.
  - ▶  $f(R_i) \succ_i f(R)$  for no  $R \in \mathcal{R}^{\mathcal{F}(\mathcal{N})}$
- **Strong participation:** Voting is **always at least as good** as abstaining.
  - ▶  $f(R) \succeq_i f(R_i)$  for all  $R \in \mathcal{R}^{\mathcal{F}(\mathcal{N})}$
- **Very strong participation:** Voting is **always better** than abstaining.
  - ▶  $f(R) \succ_i f(R_i)$  for all  $R \in \mathcal{R}^{\mathcal{F}(\mathcal{N})}$  (whenever this is possible)

**Example:** Let  $f$  return the uniform lottery over all alternatives with the highest out-degree in the majority graph.



Now, let voters 👤1, 👤2, and 👤3 join the electorate successively.

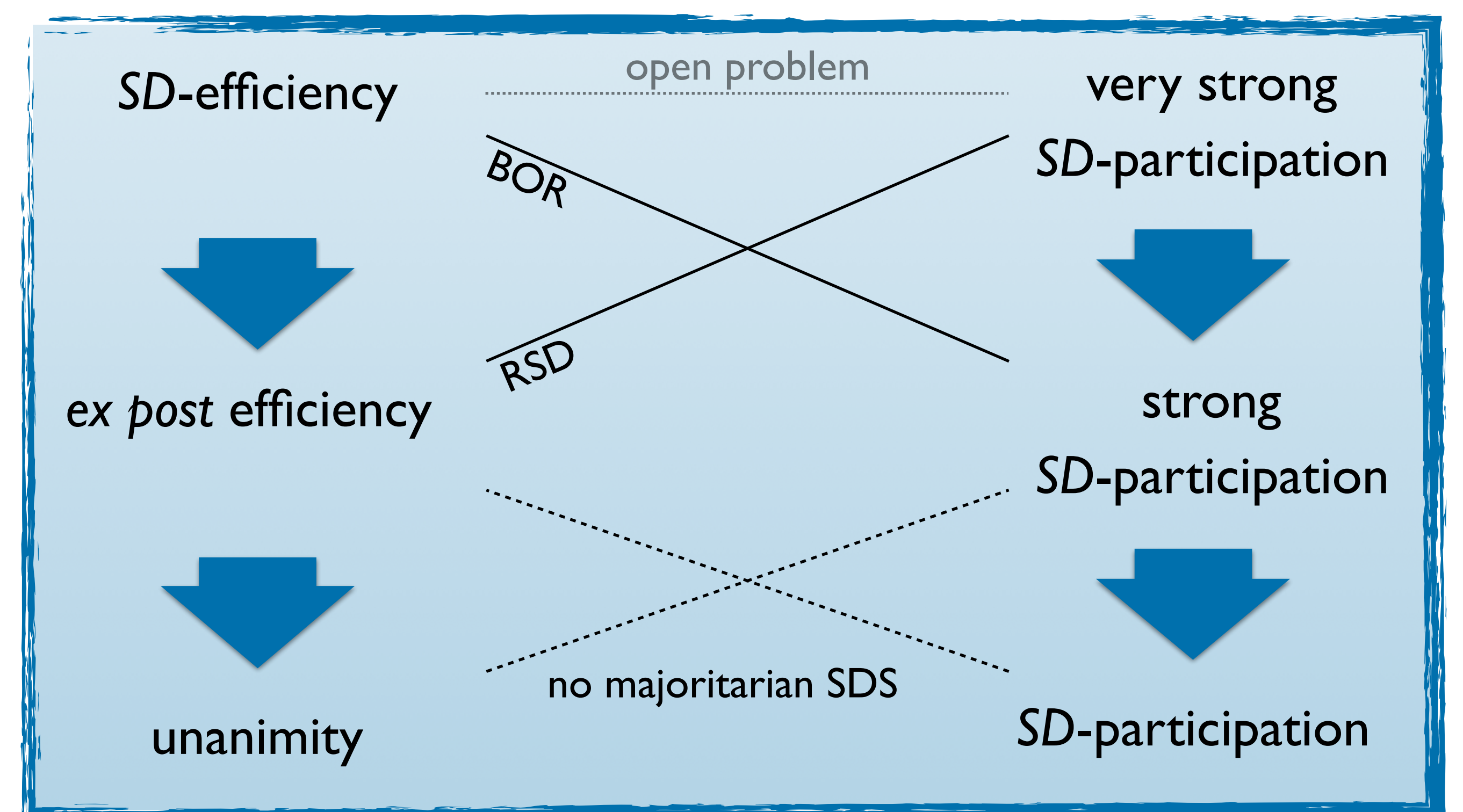


## Degrees of Efficiency

- **Unanimity:** If all agents rank the same alternative on top, it has to be selected uniquely.
- **Ex post efficiency:** Pareto-dominated alternatives have to receive probability zero.
- **SD-efficiency:** No SD-dominated lotteries may be selected.

## Main Theorems

- RSD satisfies very strong SD-participation and ex post efficiency.
- BOR satisfies strong SD-participation and SD-efficiency.
- No majoritarian SDS satisfies strong SD-participation and unanimity.
- No majoritarian SDS satisfies SD-participation and ex post efficiency.



## Summary & Outlook

- Very strong participation is prohibitive in non-probabilistic social choice but can be satisfied in probabilistic social choice.
- For further results concerning participation by groups of voters and pairwise SDSs please see the table below.
- Two interesting open problems:
  - ▶ Is there an SDS satisfying SD-efficiency and very strong SD-participation?
  - ▶ Is there a Condorcet-extension satisfying strong SD-participation?

		SD-efficient	ex post efficient	unanimous	unrestricted
very strong SD-part.	majoritarian	—	—	—	—
	pairwise	—	—	—	+
	anonymous and neutral	?	+	+	+
strong SD-part.	majoritarian	—	—	—	++
	pairwise	+	+	+	++
	anonymous and neutral	+	+	+	++
SD-part.	majoritarian	—	—	++	++
	pairwise	++	++	++	++
	anonymous and neutral	++	++	++	++

Existence of SDSs combining certain notions of efficiency and participation. + and ++ indicate the existence of SDSs satisfying single-agent and group-participation, respectively. — indicates an impossibility.