Efficiency and Strategyproofness in Randomized Social Choice

Haris Aziz Florian Brandl Felix Brandt Markus Brill Christian Geist Johannes Hofbauer

Technische Universität München

Randomized Social Choice

A social decision scheme (SDS) maps a preference profile to a lottery over alternatives. Two important properties of social decision schemes are

- **efficiency:** no agent can be made better of without making another agent worse off, and
- **strategyproofness:** no agent can obtain a more preferred outcome by misreporting his preferences.

We only consider SDS that treat all agents and all alternatives equally, i.e., SDSs that are *anonymous* and *neutral*, respectively.

Preferences over Lotteries

A lottery p is preferred to a lottery q according to

- **the sure thing (ST) extension**, if all alternatives in the support of p are preferred to all alternatives in the support of q, while ignoring alternatives that have the same probability in p and q,
- **stochastic dominance (SD)**, if, for every alternative x, p is at least as likely to yield an alternative at least as good as x as is q, and
- **pairwise comparison (PC)**, if the probability that p yields an alternative preferred to the alternative returned by q is at least as large as the other way round.

Example. Let $a \succ b \succ c$. Then,

$$\begin{aligned} &{}^2\!/_3\, a + {}^1\!/_3\, b \succ^{ST} {}^1\!/_3\, b + {}^2\!/_3\, c, \\ &{}^1\!/_3\, a + {}^2\!/_3\, b \succ^{SD} {}^1\!/_3\, a + {}^2\!/_3\, c, \text{ and} \\ &{}^2\!/_3\, a + {}^1\!/_3\, c \succ^{PC} b. \end{aligned}$$

Random Serial Dictatorship

When applying *random serial dictatorship (RSD)*, a permutation of agents is drawn uniformly at random and agents narrow down the set of alternatives in that order to their most preferred alternatives among the remaining alternatives.

R

1	2	3	4
$\{a,c\}$	$\{b,d\}$	a	b
b	a	d	c
d	c	$\{b,c\}$	$\{a,d\}$

$$RSD(R) = {}^5\!/_{12}\, a + {}^5\!/_{12}\, b + {}^1\!/_{12}\, c + {}^1\!/_{12}\, d$$

Theorem 1. *RSD is* ex post *efficient and PC-strategyproof.*

Strict Maximal Lotteries

Strict maximal lotteries (SML) correspond to the mixed quasistrict Nash equilibria of the plurality game underlying the preferences of the agents.

R

1	2	3	4	5
a	a	b	b	c
b	b	c	c	a
c	c	a	a	b

$$SML(R) = {}^3\!/_5\, a + {}^1\!/_5\, b + {}^1\!/_5\, c$$

$$\begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{ccc} a & b & c \\ \end{array}$$

$$\begin{array}{c|ccc} & a & b & c \\ \hline a & 0 & 1 & -1 \\ b & -1 & 0 & 3 \\ c & 1 & -3 & 0 \end{array}$$

Theorem 2. (Aziz et al., 2013, 2014) *SML is PC-efficient and ST-strategyproof.*

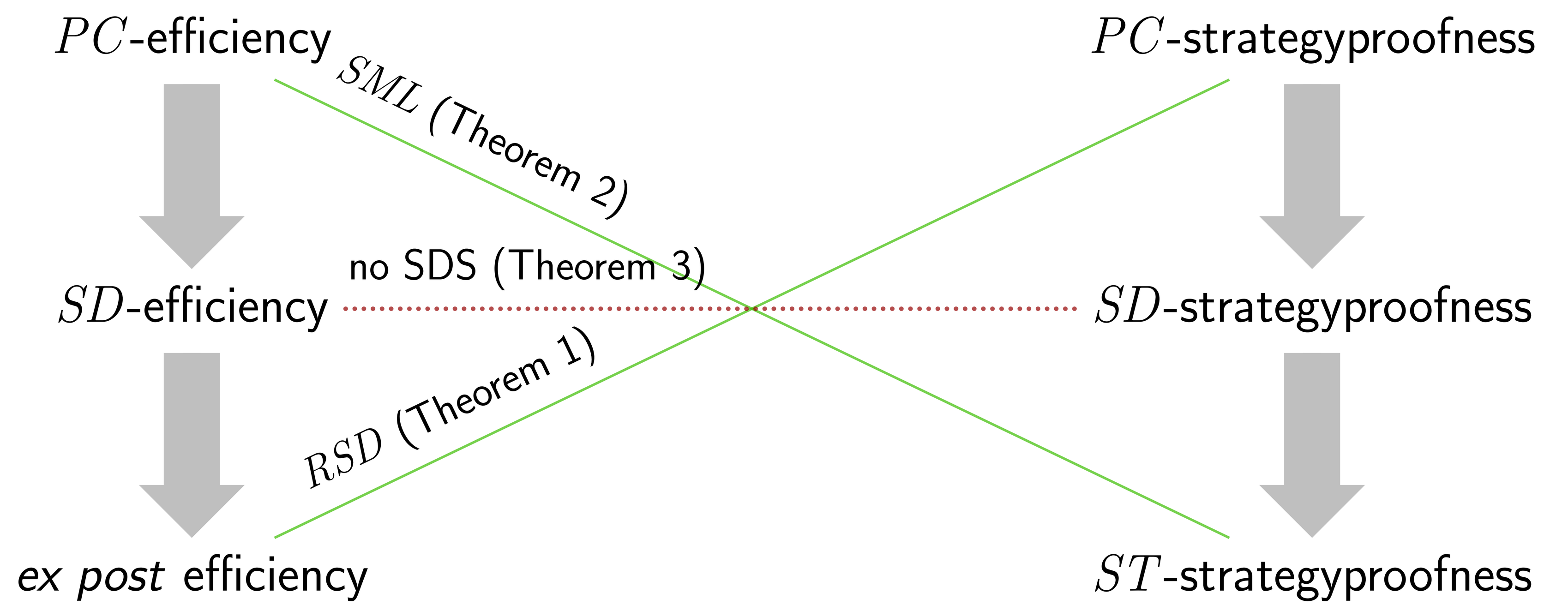
An Impossibility Theorem

We show that RSD and SML are optimal with respect to the notions of efficiency and strategyproofness considered here.

Theorem 3. (Brandl et al., 2016a) *There is no anonymous and neutral SDS that is SD-efficient and SD-strategyproof.*

This theorem generalizes previous results by Aziz et al. (2013, 2014); Brandl et al. (2016c) for the social choice domain and results by Zhou (1990); Bogomolnaia and Moulin (2001); Katta and Sethuraman (2006) for the sub-domain of house allocation if interpreted as results for the social choice domain.

This result has been derived with the help of computer aided solving techniques. The extracted proof is too long to be human readable.



Overview

	RSD	ML
SD-efficiency	✗ (only *ex post* efficiency)	✓
SD-strategyproofness	✓	✗ (only ST-strategyproofness)
SD-participation	✓ (even very strongly)	✓ (even for groups)
Condorcet consistency	✗	✓
Population consistency	✓ (only for strict preferences)	✓
Agenda consistency	✓	✓
Composition consistency	✗	✓
Efficient computability	✗	✓

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References

H. Aziz, F. Brandt, and M. Brill. On the tradeoff between economic efficiency and strategyproofness in randomized social choice. In *Proceedings of the 12th International Conference on Autonomous Agents and Multi-Agent Systems (AAMAS)*, pages 455–462. IFAAMAS, 2013.

H. Aziz, F. Brandl, and F. Brandt. On the incompatibility of efficiency and strategyproofness in randomized social choice. In *Proceedings of the 28th AAAI Conference on Artificial Intelligence (AAAI)*, pages 545–551. AAAI Press, 2014.

A. Bogomolnaia and H. Moulin. A new solution to the random assignment problem. *Journal of Economic Theory*, 100(2):295–328, 2001.

F. Brandl, F. Brandt, and C. Geist. Proving the incompatibility of efficiency and strategyproofness via SMT solving. In *Proceedings of the 25th International Joint Conference on Artificial Intelligence (IJCAI)*, 2016a. Forthcoming.

F. Brandl, F. Brandt, and H. G. Seedig. Consistent probabilistic social choice. *Econometrica*, 2016b. Forthcoming.

F. Brandl, F. Brandt, and W. Suksompong. The impossibility of extending random dictatorship to weak preferences. *Economics Letters*, 141:44–47, 2016c.

A.-K. Katta and J. Sethuraman. A solution to the random assignment problem on the full preference domain. *Journal of Economic Theory*, 131(1):231–250, 2006.

L. Zhou. On a conjecture by Gale about one-sided matching problems. *Journal of Economic Theory*, 52(1):123–135, 1990.