Incentives for Participation and Abstention in Probabilistic Social Choice*

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ABSTRACT

Voting rules are powerful tools that allow multiple agents to aggregate their preferences in order to reach joint decisions. A common flaw of some voting rules, known as the *no-show paradox*, is that agents may obtain a more preferred outcome by abstaining an election. Whenever a rule does not suffer from this paradox, it is said to satisfy *participation*. In this paper, we initiate the study of participation in *probabilistic social choice*, i.e., for voting rules that yield probability distributions over alternatives. We consider three degrees of participation based on expected utility, the strongest of which even requires that an agent is strictly better off by participating at an election. While the latter condition is prohibitive in non-probabilistic social choice, we show that it can be met by reasonable probabilistic functions. More generally, we study to which extent participation and Pareto efficiency are compatible. To the best of our knowledge, this is the first work in this direction.

Categories and Subject Descriptors

[**Theory of Computation**]: Algorithmic mechanism design; [**Theory of Computation**]: Algorithmic game theory; [**Theory of Computation**]: Randomness, geometry, and discrete structures

General Terms

Economics, Theory

Keywords

Voting theory; Pareto-optimality; no-show paradox; strategic manipulation; stochastic dominance; randomization

1. INTRODUCTION

Whenever a group of multiple agents aims at reaching a joint decision in a fair and satisfactory way, they need to aggregate their (possibly conflicting) preferences. Preference aggregation rules are studied in detail in social choice theory and are coming under increasing scrutiny from computer scientists who are interested in their computational properties or want to utilize them in computational multiagent systems.

A common flaw of many such rules, first observed by Fishburn and Brams [22], who called it the *no-show paradox*, is that agents may obtain a more preferred outcome by abstaining an election. In a seminal paper, Moulin [26] has shown that all resolute, i.e., single-valued, Condorcet extensions are susceptible to the no-show paradox. Condorcet extensions comprise a large class of voting rules that satisfy otherwise rather desirable properties.

In this paper, we initiate the study of participation in *probabilistic social choice*, i.e., we consider functions that map the ordinal preferences of the agents to a probability distribution (or lottery) over the alternatives. Gibbard [23] called these functions *decision schemes* and they are now usually referred to as social decision schemes (SDSs). Previous work on participation focused on resolute voting rules [see, e.g., 26, 32, 27, 25] and set-valued voting rules [see, e.g., 28, 24, 10]. To the best of our knowledge, this is the first time participation is explored in the context of probabilistic social choice.

Randomized voting rules have a surprisingly long tradition going back to ancient Greece and have recently gained increased attention in political science [see, e.g., 17, 35] and social choice theory [see, e.g., 9, 13]. Within computer science, randomization is a very successful technique in algorithm design and is being considered more and more often in the context of voting [see, e.g., 15, 31, 37, 34, 7, 3, 1, 4, 5].

In order to reason about the outcomes of SDSs, we need to define how agents compare lotteries when only their preferences over alternatives are known. A common assumption is that agents are equipped with von Neumann-Morgenstern (vNM) utility functions, i.e., functions that assign a cardinal utility value to each alternative. These functions are usually unknown to the social planner and may even be unknown to the agent himself. Under these assumptions, preferences over alternatives are extended to preferences over lotteries using *stochastic dominance (SD)* [see, e.g., 23, 30, 8]. One lottery stochastically dominates another if, for every alternative x, the former is at least as likely to yield an alternative at least as good as x as the latter. It is a well-known fact that a lottery stochastically dominates another if the former yields at least as much expected utility as the latter for every vNM function that is compatible with the ordinal preferences over alternatives.

*Modified in August 2018. The previous version contained an incorrect example of a pairwise SDS that satisfies very strong participation (Theorem 2).

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Based on stochastic dominance, we introduce three novel notions of participation that form a hierarchy. The weakest notion is *SD-participation.* It prescribes that no agent can obtain an *SD*-preferred outcome by leaving the electorate. In game-theoretic terms, an SDS satisfies *SD*-participation if voting is strictly undominated (by not voting). Since the preferences over lotteries obtained from stochastic dominance are incomplete, *SD*-participation does not rule out that an agent obtains an incomparable lottery by leaving the electorate. Hence, according to his (unknown) vNM function, he may receive higher expected utility by abstaining. *Strong SD-participation* requires that, for every compatible vNM function, an agent has to receive at least as much expected utility by voting than by not voting, i.e., voting yields a weakly *SD*-preferred lottery compared to not voting. In game-theoretic terms, strong *SD*-participation demands that voting is a weakly dominant strategy.

Still, in many cases, a single agent may not be able to affect the election outcome by participating. Hence, if there is some marginal cost involved in casting one's ballot, agents are better off by not voting. This concern is annihilated by *very strong SD-participation* which requires that an agent always obtains a strictly *SD*-preferred outcome by voting (unless his most preferred outcome is already chosen). In game-theoretic terms, voting is a strictly dominant strategy (whenever this is possible). Very strong participation is a remarkably strong property that cannot be met by reasonable non-probabilistic voting rules. In fact, the common phenomenon that individual agents usually face no incentive to vote because they cannot affect the outcome at all is sometimes dubbed the *Downs paradox* or the *paradox of voting.* Interestingly, we find that very strong participation can be satisfied *ex ante* by reasonable SDSs in probabilistic social choice, i.e., agents do face a (potentially very small) incentive to increase their *expected* utility by voting.

Also for the weaker notions of participation, we obtain more positive results than in the non-probabilistic setting. In contrast to Moulin's negative result for resolute Condorcet extensions mentioned at the beginning of this section, there are probabilistic Condorcet extensions that satisfy participation.

More generally, this paper studies to which extent participation is compatible with various notions of (Pareto) efficiency. We consider *SD-efficiency*—improving the satisfaction of one agent with respect to stochastic dominance will hurt another agent, *ex post* efficiency—Pareto-dominated alternatives receive probability zero, and *unanimity*—any alternative that is uniquely top-ranked by all agents will be selected with probability one.

Some of our results pertain to limited classes of SDSs. In particular, we consider pairwise SDSs and majoritarian SDSs that only take into account the weighted and unweighted majority comparisons between pairs of alternatives, respectively. Our impossibility results are as follows.

- Very strong *SD*-participation cannot be satisfied by majoritarian SDSs (Theorem 1).
- Very strong *SD*-participation is incompatible with unanimity for pairwise SDSs (Theorem 3).
- Strong *SD*-participation is incompatible with unanimity for majoritarian SDSs (Theorem 6).
- *SD*-participation is incompatible with *ex post* efficiency for majoritarian SDSs (Theorem 8).

We also obtain two positive results.

- There are SDSs that satisfy very strong *SD*-participation and *ex post* efficiency (Theorem 4).
- There are pairwise SDSs that satisfy strong *SD*-participation and *SD*-efficiency (Theorem 7).

2. RELATED WORK

Fishburn and Brams [22] first observed that agents who share the same least-preferred alternative can make exactly this alternative win by joining the electorate under the single transferable vote (STV) rule and referred to this phenomenon as the *no-show paradox.* Ray [32] showed that the *no-show paradox* for STV is likely to occur in practice. Moulin [26] defined *participation* as the property that requires that no agent is ever worse off by joining an electorate and proved that all resolute Condorcet extensions fail to satisfy participation. Participation was studied in more detail by Pérez [27] and Lepelley and Merlin [25]. However, all these papers consider resolute, i.e., single-valued, voting rules. Pérez [28], Jimeno et al. [24], and Brandt [10] have obtained various results on participation for set-valued voting rules using differing assumptions on how agents compare sets of alternatives. Abstention in slightly different contexts than the one studied in this paper has recently also caught the attention of computer scientists working on voting equilibria and campaigning [see, e.g., 16, 6]. To our best knowledge, participation has never been considered in the context of probabilistic social choice.

Participation is similar to, but logically independent from, strategyproofness. An SDS is strategyproof if no agent can obtain a more preferred outcome by misrepresenting his preferences. Recently, a number of theorems that illustrate the tradeoff between strategyproofness and efficiency in probabilistic social choice have been shown [8, 9, 3, 4]. Bogomolnaia and Moulin [8] have proved that strong *SD*-strategyproofness and *SD*-efficiency are incompatible. Aziz et al. [3] conjectured that this incompatibility even holds for (weak) *SD*-strategyproofness and proved this claim for majoritarian SDSs. This statement was later strengthened to pairwise SDSs [4]. We show in this paper that no corresponding statement for participation holds by proposing an SDS that satisfies strong *SD*-participation, *SD*-efficiency, and even pairwiseness (Theorem 7).

Manipulation by abstention is arguably a more severe problem than manipulation by misrepresentation for two reasons. First, agents might not be able to find a successful strategic misrepresentation of their preferences. It was shown in various papers that the corresponding computational problem can be intractable [see, e.g., 20]. Finding a successful manipulation by strategic abstention, on the other hand, is never harder than computing the outcome of the voting rule. Secondly, one could argue that agents will not lie about their preferences because this is considered immoral (Borda famously exclaimed "my scheme is intended only for honest men"), while strategic abstention is deemed acceptable.[1]

[1] Alternatively, one could also argue that manipulation by misrepresentation is more critical because agents are

3. PRELIMINARIES

Let A be a finite set of alternatives and $\mathbb{N} = \{1, 2, \dots\}$ a set of agents. $\mathcal{F}(\mathbb{N})$ denotes the set of all finite and non-empty subsets of $\mathbb{N}$. A *(weak) preference relation* is a complete, reflexive, and transitive binary relation on A. The preference relation of agent i is denoted by R_i. The set of all preference relations is denoted by $\mathcal{R}$. In accordance with conventional notation, we write P_i for the strict part of R_i, i.e., $x \; P_i \; y$ if $x \; R_i \; y$ but not $y \; R_i \; x$ and I_i for the indifference part of R_i, i.e., $x \; I_i \; y$ if $x \; R_i \; y$ and $y \; R_i \; x$. A preference relation R_i is *linear* if $x \; P_i \; y$ or $y \; P_i \; x$ for all distinct alternatives $x, y \in A$. For $X \in \mathcal{F}(\mathbb{N})$, we denote by $\max_{R_i}(X)$ the set of agent i's most preferred alternatives in X, i.e., $\max_{R_i}(X) = \{x \in X \colon x \; R_i \; y \text{ for all } y \in X\}$. We will compactly represent a preference relation as a comma-separated list with all alternatives among which an agent is indifferent placed in a set. For example $x \; P_i \; y \; I_i \; z$ is represented by $R_i \colon x, \{y, z\}$.

A *preference profile* R is a function from a set of agents N to the set of preference relations $\mathcal{R}$. The set of all preference profiles is denoted by $\mathcal{R}^{\mathcal{F}(\mathbb{N})}$. For a preference profile $R \in \mathcal{R}^N$ and $S \subseteq N$, $T \subseteq \mathbb{N}$, $i \in N$, $j \in \mathbb{N}$ we define

$$R_{-i} = R \setminus \{(i, R_i)\}, \qquad R_{+j} = R \cup \{(j, R_j)\},$$
$$R_{-S} = R \setminus \bigcup_{k \in S} \{(k, R_k)\}, \text{ and } \qquad R_{+T} = R \cup \bigcup_{k \in T} \{(k, R_k)\}.$$

By $n_R(x, y)$ we denote the number of agents who weakly prefer x to y, i.e., $n_R(x, y) = |\{i \in N \colon x \; R_i \; y\}|$. Whenever R is clear from the context we only write $n(x, y)$. In addition, we define the *majority margin* $g_R(x, y)$ as $g_R(x, y) = n_R(x, y) - n_R(y, x)$. The *majority relation* R_M of a preference profile R is given by the majority comparisons between each pair of alternatives, i.e., $x \; R_\mathrm{M} \; y$ iff $n(x, y) \geq n(y, x)$. An alternative x is a *Condorcet winner* if $x \; P_\mathrm{M} \; y$ for all $y \in A \setminus \{x\}$.

Let furthermore $\Delta(A)$ denote the set of all *lotteries* (or probability distributions) over A, i.e.,

$$\Delta(A) = \left\{ \sum_{x \in A} p(x) \cdot x \colon \; \sum_{x \in A} p(x) = 1, \forall x \in A \colon p(x) \geq 0 \right\}.$$

We usually write lotteries as convex combinations of alternatives, i.e., $1/2\, a + 1/2\, b$ denotes the uniform distribution over $\{a, b\}$. For a lottery $p \in \Delta(A)$ and an alternative $x \in A$, $p(x)$ denotes the probability that p assigns to x. Similarly, for a set of alternatives $S \in A$, $p(S)$ denotes the sum of probabilities that p assigns to alternatives in S. The *support* of p, denoted $\mathrm{supp}(p)$, is the set of all alternatives to which p assigns positive probability, i.e., $\mathrm{supp}(p) = \{x \in A \colon p(x) > 0\}$.

Our central objects of study are *social decision schemes (SDSs)*, i.e., functions that map a preference profile to a lottery. Formally, an SDS is a function $f \colon \mathcal{R}^{\mathcal{F}(\mathbb{N})} \to \Delta(A)$. A social choice function (SCF), on the other hand, is a function $f \colon \mathcal{R}^{\mathcal{F}(\mathbb{N})} \to 2^A \setminus \emptyset$ that maps a preference profile to a subset of alternatives. A minimal fairness condition for SDSs (and SCFs) is *anonymity*, which requires that $f(R) = f(R')$ for all $N, M \in \mathcal{F}(\mathbb{N})$, $R \in \mathcal{R}^N$, $R' \in \mathcal{R}^M$, and bijections $\pi \colon N \to M$ such that $R'_i = R_{\pi(i)}$ for all $i \in N$. Another fairness requirement is *neutrality*. For a permutation π of A and a preference relation R_i, define $\pi(x) \; R_i^\pi \; \pi(y)$ if and only if $x \; R_i \; y$. Then, an SDS (or SCF) f is *neutral* if for all permutations π and all $R \in \mathcal{R}^{\mathcal{F}(\mathbb{N})}$, $f(R)(x) = f(R^\pi)(\pi(x))$ for all $x \in A$.

An SDS (or SCF) f is *pairwise* if it is neutral and for all preference profiles $R, R' \in \mathcal{R}^{\mathcal{F}(\mathbb{N})}$, $f(R) = f(R')$ whenever $g_R(x, y) = g_{R'}(x, y)$ for all alternatives x, y.[2] In other words, the outcome of a pairwise SDS only depends on the anonymized comparisons between pairs of alternatives [see, e.g., 38, 39]. Many common SCFs are pairwise [see, e.g., 21]. Typical examples are Borda's rule, Kemeny's rule, or the Simpson-Kramer rule (aka maximin).

An SDS (or SCF) f is *majoritarian* (or a neutral C1 function) if it is neutral and for all preference profiles $R, R' \in \mathcal{R}^{\mathcal{F}(\mathbb{N})}$, $f(R) = f(R')$ whenever $R_\mathrm{M} = R'_\mathrm{M}$. Even the seemingly narrow class of majoritarian SCFs contains a variety of functions (sometimes called *tournament solutions*). Examples include Copeland's rule, the top cycle, or the uncovered set [see, e.g., 11].

It is easy to see that the three classes form a hierarchy: every majoritarian SDS is pairwise and every pairwise SDS is anonymous and neutral.

An SDS is a *Condorcet extension* if it puts probability one on a Condorcet winner whenever one exists. Except Borda's rule, all of the pairwise and majoritarian SDSs mentioned above are Condorcet extensions.

In order to reason about the outcomes of SDSs, we need to make assumptions on how agents compare lotteries. A common way to extend preferences over alternatives to preferences over lotteries is *stochastic dominance (SD)*. A lottery *SD*-dominates another if, for every alternative x, the former is at least as likely to yield an alternative at least as good as x as the latter. Formally,

$$p \; R_i^{SD} \; q \text{ iff for all } x \in A, \sum_{y \colon y R_i x} p(y) \geq \sum_{y \colon y R_i x} q(y).$$

It is well-known that $p \; R_i^{SD} \; q$ iff the expected utility for p is at least as large as that for q for every vNM function compatible with R_i.

Thus, for the preference relation $R_i \colon a, b, c$, we for example have that

$$(2/3\, a + 1/3\, c) \; P_i^{SD} \; (1/3\, a + 1/3\, b + 1/3\, c)$$

while $2/3\, a + 1/3\, c$ and b are incomparable.

4. EFFICIENCY AND PARTICIPATION

In this section we define the notions of efficiency and participation considered in this paper. The three notions of efficiency defined below are generalizations of established efficiency notions for SCFs. A rather basic requirement is *unanimity* [see e.g., 33, 29, 13]. For SCFs, unanimity prescribes that if all agents report the same alternative as their (unique) top choice, this alternative is chosen uniquely. The arguably most natural generalization of unanimity to probabilistic social choice is that if all agents report the same alternative as their top choice, this alternative is chosen with probability one.

An alternative is *Pareto-dominated* if there exists another alternative such that all agents weakly prefer the latter to the former with a strict preference for at least one agent.

tempted to act immorally, which is a valid, but different, concern.

[2] Apart from some technical subtleties, this is what Fishburn calls C2 functions [21].

An SDS is *ex post efficient* if it assigns probability zero to all Pareto-dominated alternatives [see e.g., 23, 9, 19].

Finally, we define efficiency with respect to stochastic dominance. A lottery p is *SD-efficient* if there is no other lottery q that is weakly preferred by every agent with a strict preference for at least one agent, i.e., $q\ R_i^{SD}\ p$ for all $i \in N$ and $q\ P_i^{SD}\ p$ for some $i \in N$. An SDS is *SD*-efficient if it returns an *SD*-efficient lottery for every preference profile [see e.g., 8, 4, 5].

These notions of efficiency form a hierarchy (see Figure 1). It is well-known that *SD*-efficiency implies *ex post* efficiency and from the definitions it follows straightforwardly that *ex post* efficiency implies unanimity. Moreover, it is easily seen that every Condorcet extension satisfies unanimity.

For better illustration consider $A = \{a, b, c, d\}$ and the preference profile $R = (R_1, \dots, R_4)$,

$$R_1\colon a, c, b, d, \qquad R_2\colon b, d, a, c,$$
$$R_3\colon a, d, b, c, \qquad R_4\colon b, c, a, d.$$

Observe that no alternative is Pareto-dominated, i.e., for instance the uniform lottery $1/4\, a + 1/4\, b + 1/4\, c + 1/4\, d$ is *ex post* efficient. On the other hand, the uniform lottery is not *SD*-efficient as all agents strictly *SD*-prefer $1/2\, a + 1/2\, b$.

Participation prescribes that no agent can obtain a more preferred outcome by abstaining the election. We obtain varying degrees of this property associated with stochastic dominance based on the interpretation of incomparabilities and ties (see Figure 1). The weakest notion of participation we consider is *SD*-participation, i.e., no agent can obtain an *SD*-preferred outcome by not voting. Formally, an SDS f is *SD*-manipulable (by strategic abstention) if there exist $R \in \mathcal{R}^N$ for some $N \in \mathcal{F}(\mathbb{N})$ and $i \in N$ such that $f(R_{-i})\ P_i^{SD}\ f(R)$. If an SDS is not *SD*-manipulable it satisfies *SD-participation*.

However, it may be interpreted as a successful manipulation by abstention if an agent can obtain a lottery that is incomparable (according to stochastic dominance) to the lottery he obtains by voting, since the former yields more expected utility than the latter for *some* (rather than all) compatible vNM functions. Strong *SD*-participation requires that voting is a weakly dominant strategy. Formally, an SDS f satisfies *strong SD-participation* if $f(R)\ R_i^{SD}\ f(R_{-i})$ for all $N \in \mathcal{F}(\mathbb{N})$, $R \in \mathcal{R}^N$, and $i \in N$.

We obtain the strongest notion of participation considered in this paper by requesting that voting is a strictly dominant strategy (whenever this is possible). An SDS f satisfies *very strong SD-participation* if for all $N \in \mathcal{F}(\mathbb{N})$, $R \in \mathcal{R}^N$, and $i \in N$, $f(R)\ R_i^{SD}\ f(R_{-i})$ and

$$f(R)\ P_i^{SD}\ f(R_{-i}) \text{ whenever } \exists p \in \Delta(A)\colon p\ P_i^{SD}\ f(R_{-i})$$

Hence, voting has to be a strictly dominant strategy for every agent unless he already receives one of his most preferred outcomes when abstaining.

By the same token, we obtain three notions of *group-participation*. Note that *SD*-group-manipulability requires *all* agents to be strictly better off after having left the electorate.

5. RESULTS AND DISCUSSION

In the following, we look at the different notions of participation introduced above together with varying types of efficiency and fairness.

5.1 Very Strong *SD*-participation

Very strong *SD*-participation is the strongest participation-property considered in this paper. As every single agent who joins an electorate has to be able to influence the outcome in his favor, it can be easily seen that no majoritarian SDS can satisfy this property.

THEOREM 1. *There is no majoritarian SDS satisfying very strong SD-participation even when preferences are required to be linear.*

PROOF. Let $R = (R_1, R_2, R_3)$ such that $R_1, R_2\colon a, b$, $R_3\colon b, a$. Note that $R_\mathrm{M} = (R_{-3})_\mathrm{M}$. Therefore, for every majoritarian SDS f it holds that $f(R) = f(R_{-3})$ giving that it never can be the case that $f(R)\ P_3^{SD}\ f(R_{-3})$. □

In comparison to majoritarian SDSs, pairwise SDSs allow for more sensitivity with respect to variations in the set of agents. Still, when additionally requiring unanimity, we obtain another impossibility.

THEOREM 3. *There is no pairwise and unanimous SDS satisfying very strong SD-participation even when preferences are required to be linear.*

PROOF. Let $R = (R_1, R_2, R_3)$ be a preference profile such that $R_1, R_2\colon a, b$, $R_3\colon b, a$. Note that any pairwise and unanimous SDS f has to choose $f((R_1, R_2)) = f((R_1)) = a$ and by the fact that $g_{(R_1)}(a, b) = g_R(a, b)$ we get $f((R_1)) = f(R)$. Put together, it can never be the case that $f(R)\ P_3^{SD}\ f(R_{-3})$. □

Within the unrestricted domain of SDSs, there do exist functions that satisfy very strong *SD*-participation and certain notions of efficiency, in particular *random serial dictatorship (RSD)*—the canonical generalization of random dictatorship [see e.g., 23] to weak preferences. *RSD* is defined by picking a sequence of the agents uniformly at random and then letting each agent narrow down the set of alternatives by picking his most preferred of the alternatives selected by the previous agents. Formally, we obtain the following recursive definition.

$$RSD(R, X) = \begin{cases} \sum_{x \in X} \frac{1}{|X|}\ x & \text{if } R = \emptyset, \\ \sum_{i=1}^{|R|} \frac{1}{|R|}\ RSD(R_{-i}, \max_{R_i}(X)) & \text{otherwise,} \end{cases}$$

and $RSD(R) = RSD(R, A)$. For a definition of *RSD* employing permutations we refer to Aziz et al. [4].

THEOREM 4. *RSD satisfies anonymity, neutrality,* ex post *efficiency, and very strong SD-participation.*

PROOF. We only prove very strong *SD*-participation here and refer to Aziz et al. [3] for *ex post* efficiency. Let $N \in \mathcal{F}(\mathbb{N})$, $R \in \mathcal{R}^N$, and $i \in N$. A first step for showing very strong *SD*-participation is to prove that $RSD(R)\ R_i^{SD}\ RSD(R_{-i})$. It is already known that *RSD* satisfies *strong SD-strategyproofness*, i.e., $RSD(R)\ R_i^{SD}\ RSD(R')$ for every preference profile $R' \in \mathcal{R}^N$ where $R'_j = R_j$ for all $j \neq i$ [see, e.g., 3]. If agent i is completely indifferent between all alternatives in A, it trivially holds that $RSD(R) = RSD(R_{-i})$. We obtain as a direct consequence that *RSD* satisfies strong *SD*-participation.

In order to see that the even stronger notion applies, assume that R_{-i} allows for a strict improvement for i, i.e.,

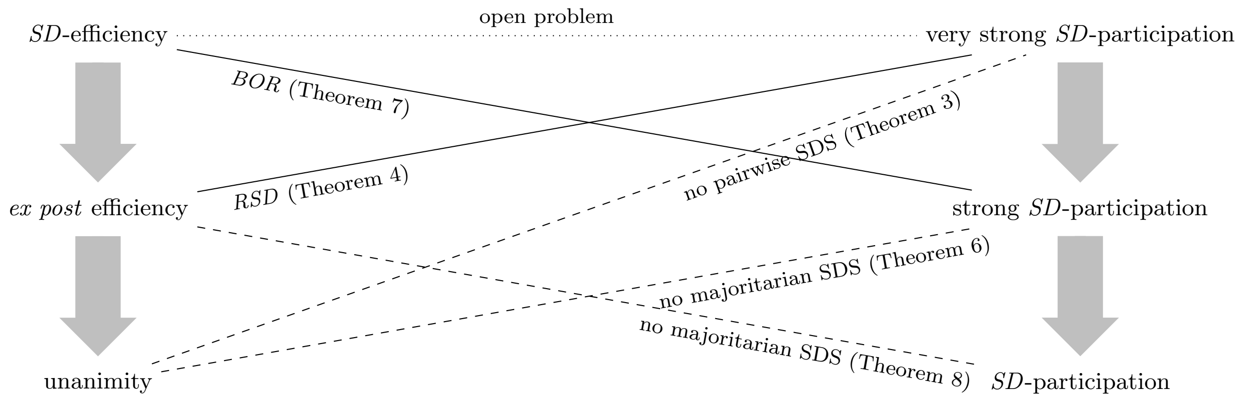


Figure 1: Relationships between efficiency and participation concepts. An arrow from one notion of efficiency or participation to another denotes that the former implies the latter. A solid line indicates that there exist SDSs with the given properties. A dashed line indicates that no SDSs with the given properties exists. The dotted line marks an open problem.

there is $p \in \Delta(A)$ such that $p\ P_i^{SD}\ RSD(R_{-i})$. Thus, $RSD(R_{-i})(\max_{R_i}(A)) < 1$. We have

$$\begin{aligned}
&RSD(R)(\max_{R_i}(A)) = RSD(R,A)(\max_{R_i}(A)) \\
&= \sum_{j\in N} \frac{1}{n}\ RSD(R_{-j}, \max_{R_j}(A))(\max_{R_i}(A)) \\
&= \frac{1}{n} + \frac{1}{n} \sum_{j\in N\setminus\{i\}} \underbrace{RSD(R_{-j}, \max_{R_j}(A))(\max_{R_i}(A))}_{\geq RSD(R_{-\{i,j\}}, \max_{R_j}(A))(\max_{R_i}(A))} \\
&\geq \frac{1}{n} + \frac{n-1}{n} \underbrace{RSD(R_{-i}, A)(\max_{R_i}(A))}_{<1 \text{ by assumption}} \\
&> RSD(R_{-i}, A)(\max_{R_i}(A)) = RSD(R_{-i})(\max_{R_i}(A)).
\end{aligned}$$

We conclude that $RSD(R)\ P_i^{SD}\ RSD(R_{-i})$ for all $i \in N$ which means RSD satisfies very strong SD-participation. □

It is noteworthy that RSD does not even satisfy SD-group-participation. This can for instance be observed when looking at $A = \{a, b, c, d\}$, $R = (R_1, R_2, R_3, R_4)$, with

$$R_1\colon \{a,d\}, b, c, \qquad R_2\colon \{b,c\}, a, d,$$
$$R_3\colon \{a,c\}, b, d, \qquad R_4\colon \{b,d\}, a, c.$$

Here, we have $RSD(R) = {}^1\!/_3\, a + {}^1\!/_3\, b + {}^1\!/_6\, c + {}^1\!/_6\, d$ and $RSD(R_{-\{1,2\}}) = {}^1\!/_2\, a + {}^1\!/_2\, b$. Yet, for both agents $i \in \{1, 2\}$, it holds that $RSD(R_{-\{1,2\}})\ P_i^{SD}\ RSD(R)$.

While RSD satisfies *ex post* efficiency and the strongest notion of participation considered in this paper, it was recently shown that computing RSD is #P-complete [2].

Note that RSD is by far not the only SDS satisfying very strong SD-participation and *ex post* efficiency. Further SDSs that satisfy these properties can be obtained by taking the convex combination of RSD and other SDSs.

Theorem 5. *Let f_1, f_2 be two* ex post *efficient SDSs such that f_1 satisfies very strong SD-participation and f_2 satisfies strong SD-participation. Moreover, let $\lambda \in (0, 1)$. Then, $f = \lambda f_1 + (1-\lambda) f_2$ satisfies* ex post *efficiency and very strong SD-participation.*

Proof. Let f_1, f_2, f, and λ be as above. First note that if both f_1, f_2 put probability zero on all Pareto-dominated alternatives $x \in A$, so does f. Additionally, we have for all $y \in A$, $i \in N$, $l \in \{1, 2\}$

$$\sum_{x\in A\colon xR_iy} f_l(R)(x) \geq \sum_{x\in A\colon xR_iy} f_l(R_{-i})(x)$$

and for some $\bar{y} \in A$ it holds for all $i \in N$

$$\sum_{x\in A\colon xR_i\bar{y}} f_1(R)(x) > \sum_{x\in A\colon xR_i\bar{y}} f_1(R_{-i})(x).$$

We directly deduce that for all $y \in A$, $i \in N$

$$\sum_{x\in A\colon xR_iy} f(R)(x) \geq \sum_{x\in A\colon xR_iy} f(R_{-i})(x)$$

and for $\bar{y}$ also

$$\sum_{x\in A\colon xR_i\bar{y}} f(R)(x) > \sum_{x\in A\colon xR_i\bar{y}} f(R_{-i})(x).$$

Therefore, $f(R)\ P_i^{SD}\ f(R_{-i})$ for all $i \in N$. □

As a consequence, every proper convex combination of RSD and BOR (which will be defined in the next section) satisfies very strong SD-participation and *ex post* efficiency.

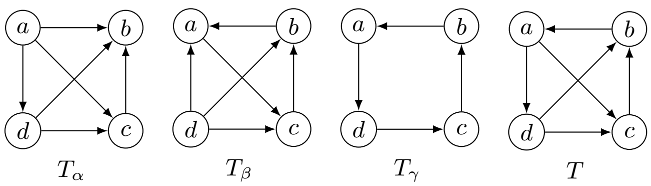
By contrast, very strong SD-participation is prohibitive in general if we consider abstention by groups of agents. More precisely, there is no SDS that satisfies very strong SD-group-participation. The proof is omitted due to space constraints.

5.2 Strong SD-participation

Since very strong SD-participation implies strong SD-participation, positive results from the previous section carry over. However, in contrast to Theorem 1, there do exist majoritarian SDSs that satisfy strong SD-participation. The arguably simplest example is a constant function that always chooses the uniform distribution over all alternatives. In the case that *ex post* efficiency is required as well, the more general impossibility shown in Theorem 8 (which only requires SD-participation) applies. With respect to unanimity, we obtain the following result.

THEOREM 6. *When $|A| \geq 4$, there is no majoritarian and unanimous SDS satisfying strong SD-participation, even when preferences are required to be linear.*

PROOF. Let $A = \{a, b, c, d\}$. For contradiction, suppose f is an SDS satisfying majoritarianness, unanimity and strong SD-participation. For the sake of readability, we slightly abuse notation and write $f(T)$ instead of $f(R)$ whenever an implication holds for all $R \in \mathcal{R}^{\mathcal{F}(\mathbb{N})}$ inducing T, i.e., $T = R_{\mathrm{M}}$. First look at the tournament T_α as depicted below and note that T_α is transitive and could thus be induced by only one agent with preferences $R_i : a, d, c, b$. By unanimity, we obtain $f(T_\alpha) = a$.



Now, let two agents α, α' with identical preferences $R_\alpha, R_{\alpha'} \colon b, a, c, d$ join an electorate with preference profile $R \in \mathcal{R}^{\mathcal{F}(\mathbb{N})}$ inducing T_α. Note that R is supposed to be of the form that $|g_R(x, y)| \geq 3$ for all $(x, y) \in A \times A \setminus \{(a, b), (b, a)\}$, $x \neq y$, and $g_R(a, b) = 1$. The additional agents alter the majority graph in a way such that it equals T for $R_{+\{\alpha,\alpha'\}}$. Using strong SD-participation, we deduce that $f(T)(c) = f(T)(d) = 0$.

As tournament T_β is the majority graph induced by $(R_{j_1}, R_{j_2}, R_{j_3})$,

$$R_{j_1} \colon d, a, c, b, \qquad R_{j_2} \colon d, b, a, c, \qquad R_{j_3} \colon d, c, b, a,$$

unanimity yields $f(T_\beta) = d$. Analogously to before, let two agents β, β' with preferences $R_\beta, R_{\beta'} \colon a, d, b, c$ join an electorate with preference profile $R' \in \mathcal{R}^{\mathcal{F}(\mathbb{N})}$ inducing T_β. We suppose R' to be of the form that $|g_{R'}(x, y)| \geq 3$ for all $(x, y) \in A \times A \setminus \{(a, d), (d, a)\}, x \neq y$ and $g_{R'}(d, a) = 1$. This changes the majority graph of R' such that it equals T for $R'_{+\{\beta,\beta'\}}$. Here, strong SD-participation gives $f(T)(b) = f(T)(c) = 0$. Consequently, we directly get $f(T) = a$.

Due to neutrality, we know that

$$f(T_\gamma) = {}^1\!/\!_4\, a + {}^1\!/\!_4\, b + {}^1\!/\!_4\, c + {}^1\!/\!_4\, d.$$

Add a single agent γ, $R_\gamma \colon d, b, a, c$ to an electorate with preference profile $R'' \in \mathcal{R}^{\mathcal{F}(\mathbb{N})}$ inducing T_γ where $g_{R''}(a, d) = g_{R''}(d, c) = g_{R''}(c, b) = g_{R''}(b, a) \geq 2$. The majority graph is thus altered such that it equals T for $R''_{+\gamma}$. Note that for γ, $f(T)$ and $f(T_\gamma)$ are incomparable according to the SD-extension contradicting that f satisfies strong SD-participation and concluding the proof. $\square$

As a corollary of this theorem, there is no majoritarian Condorcet extension satisfying strong SD-participation.

While *RSD* satisfies very strong SD-participation (Theorem 4), it fails to satisfy SD-efficiency for weak preferences [see, e.g., 8, 3]. We now define an SDS that satisfies SD-efficiency, strong SD-participation, and pairwiseness. The SDS, called *BOR*, yields the uniform distribution over all Borda winners.

For a preference profile $R \in \mathcal{R}^N$ define the Borda score of alternative x as

$$s_R(x) = \sum_{i \in N} |\{y \in A \colon x \; P_i \; y\}| + {}^1\!/\!_2\, |\{y \in A \setminus \{x\} \colon x \; I_i \; y\}|$$

A Borda winner is an alternative with the highest Borda score. *BOR* is defined as the SDS, that returns the uniform distribution over all Borda winners, i.e.,

$$BOR(R) = \frac{1}{|\arg\max_y s_R(y)|} \sum_{x \in \arg\max_y s_R(y)} x.$$

If preferences are linear, this definition of $s_R(x)$ coincides with the standard definition of Borda scores. Whenever an agent is indifferent between some alternatives $X \subseteq A$, the scores that would have been awarded to those alternatives had they been ranked linearly are summed up and equally divided among them.

THEOREM 7. *BOR satisfies pairwiseness, SD-efficiency, and strong SD-participation.*

PROOF. First we show that *BOR* is pairwise. Observe that $|\{i \in N \colon x \; I_i \; y\}| = n(x, y) + n(y, x) - n$. Rewriting the definition of $s_R(x)$ and using the fact stated before yields

$$\begin{aligned} s_R(x) &= \sum_{y \in A \setminus \{x\}} |\{i \in N \colon x \; P_i \; y\}| + {}^1\!/\!_2\, |\{i \in N \colon x \; I_i \; y\}| \\ &= \sum_{y \in A \setminus \{x\}} n(x, y) - {}^1\!/\!_2\, (n(x, y) + n(y, x) - n) \\ &= {}^1\!/\!_2\, n + {}^1\!/\!_2 \sum_{y \in A \setminus \{x\}} (n(x, y) - n(y, x)). \end{aligned}$$

Hence, the order of the $s_R(x)$ and thus also the outcome of *BOR* only depends on $n(x, y) - n(y, x)$. Consequentially, *BOR* is pairwise.

In order to see that *BOR* satisfies strong SD-participation consider the following: if by joining an electorate N_{-i}, some agent i can force an alternative a into the set of Borda winners without dropping any other, then a has to be ranked above the other Borda winners in R_i, i.e., $BOR(R) \; R_i^{SD} \; BOR(R_{-i})$. On the other hand, if i can force an alternative b out of the set of Borda winners by participating, b has to be ranked below the other Borda winners in R_i giving once more $BOR(R) \; R_i^{SD} \; BOR(R_{-i})$. A combination of both arguments yields that even for some alternatives joining the set of Borda winners as well as others leaving when i participates at the election, we always get $BOR(R) \; R_i^{SD} \; BOR(R_{-i})$.

Finally, suppose *BOR* does not satisfy SD-efficiency, i.e., there exists some electorate $N \in \mathcal{F}(\mathbb{N})$, preference profile $R \in \mathcal{R}(A)^N$, and lottery $p \in \Delta(A)$ such that $p \; R_i^{SD} \; BOR(R)$ for all $i \in N$. First, note that therefore

$$\sum_{x \in \mathrm{supp}(p)} p(x) s_R(x) \geq \sum_{x \in \mathrm{supp}(BOR(R))} BOR(R)(x) s_R(x).$$

Stated differently and taking into account that one agent has to strictly SD-prefer p, the (weighted) average Borda score of the alternatives in $\mathrm{supp}(p)$ would have to be greater than the one of the alternatives in $\mathrm{supp}(BOR(R))$ contradicting the fact that *BOR* chooses the alternatives with maximal Borda score. This concludes the proof of the theorem. $\square$

With the same proof as for Theorem 7 it can be shown that randomizing uniformly over the winners of any scoring rule with a strictly monotonic decreasing score vector satisfies SD-efficiency and strong SD-participation. Also note that *BOR* satisfies SD-group-participation. The proof uses average Borda scores within groups and works analogously. The stronger notion of strong *SD*-group-participation cannot be satisfied by any anonymous, neutral, and unanimous SDS.

5.3 *SD*-participation

In contrast to strong *SD*-participation, *SD*-participation allows for majoritarian and efficient SDSs. Interestingly, this does not only hold for single agents but for groups of agents as well.

The SDS that returns a Condorcet winner whenever one exists and the uniform lottery over all alternatives otherwise is both majoritarian and unanimous and additionally satisfies *SD*-group-participation. We find that it is not possible to further strengthen the degree of efficiency to *ex post* efficiency without losing either *SD*-participation or majoritarianness.

In order to simplify the proof of Theorem 8, we introduce some additional notation and state an auxiliary lemma linking *ex post* efficiency and participation to the (McKelvey) uncovered set [see 18]. We say that an alternative x *(McKelvey) covers* an alternative y if x is at least as good as y compared to every other alternative. Formally, x covers y if $x \; P_{\mathrm{M}} \; y$ and, for all $z \in A$, both $y \; R_{\mathrm{M}} \; z$ implies $x \; R_{\mathrm{M}} \; z$, and $z \; R_{\mathrm{M}} \; x$ implies $z \; R_{\mathrm{M}} \; y$. The *uncovered set* of R_{M}, denoted $UC(R_{\mathrm{M}})$, is the set of all alternatives that are not covered by any other alternative. By definition, UC is a majoritarian SCF.

Brandt et al. [12] have shown that for all preference profiles $R \in \mathcal{R}^{\mathcal{F}(\mathbb{N})}$ and alternatives $x, y \in A$, if x covers y in R, then there is a preference profile $R' \in \mathcal{R}^{\mathcal{F}(\mathbb{N})}$ such that $R'_{\mathrm{M}} = R_{\mathrm{M}}$ and x Pareto-dominates y in R'. Hence, every majoritarian and Pareto-optimal SCF has to select a subset of the uncovered set. Or, phrased in terms of probabilistic social choice, every majoritarian and *ex post* efficient SDS has to put all probability on alternatives in the uncovered set.

LEMMA 1. *Let f be a majoritarian and* ex post *efficient SDS. Then,* $\mathrm{supp}(f(R)) \subseteq UC(R_{\mathrm{M}})$ *for all* $R \in \mathcal{R}^{\mathcal{F}(\mathbb{N})}$.

With Lemma 1 at hand, we can now continue with our main theorem regarding the compatibility of efficiency and participation for majoritarian SDSs.

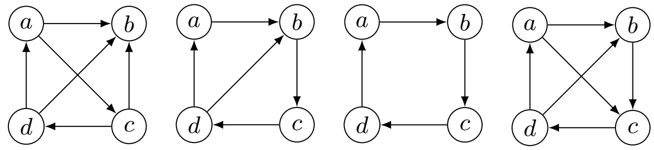
THEOREM 8. *When $|A| \geq 4$, there is no majoritarian SDS satisfying* ex post *efficiency and SD-participation.*

PROOF. Let $A = \{a, b, c, d\}$. Analogously to the proof of Theorem 6, we slightly abuse notation for the sake of readability and write $f(T)$ instead of $f(R)$ whenever an implication holds for all $R \in \mathcal{R}^{\mathcal{F}(\mathbb{N})}$ inducing T, i.e., $T = R_{\mathrm{M}}$. In order to show Theorem 8 assume for contradiction there exists a majoritarian SDS f satisfying both *ex post* efficiency and *SD*-participation. First note that by Lemma 1, alternatives not in $UC(R_{\mathrm{M}})$ receive probability zero in $f(R)$.

Start the proof by examining tournaments of the structure T as depicted below. Therefore, we begin with a preference profile $R \in \mathcal{R}^{\mathcal{F}(\mathbb{N})}$ inducing a majority graph of type T_α. As b is covered (e.g., by a), due to neutrality, f has to yield the lottery

$$f(T_\alpha) = {}^1\!/_3\, a + {}^1\!/_3\, c + {}^1\!/_3\, d.$$

Suppose this preference profile R is of the form that $g_R(a, d) = 1$ and $|g_R(x, y)| \geq 3$ for all $(x, y) \in A \times A \setminus \{(a, d), (d, a)\}, x \neq y$.



T_α T_β T_γ T

Thus, two subsequently added agents α, α' endowed with preferences $b \; P_\alpha \; c$ and $b \; P_{\alpha'} \; c$ lead to the preference profile $R_{+\{\alpha,\alpha'\}}$. The majority graph of $R_{+\{\alpha,\alpha'\}}$ is of type T and alternative b remains covered by a. This is the case independent of all other preferences, and, as f satisfies *SD*-participation, it cannot be that $f(R) \; P_\alpha^{SD} \; f(R_{+\alpha})$ or $f(R_{+\alpha}) \; P_{\alpha'}^{SD} \; f(R_{+\{\alpha,\alpha'\}})$. Due to the flexibility left in R_α and $R_{\alpha'}$, we obtain

$$f(T) = f(T_\alpha).$$

Continue with some preference profile $R' \in \mathcal{R}^{\mathcal{F}(\mathbb{N})}$ that induces a majority graph of type T_β. Without loss of generality, let $|g_{R'}(x, y)| \geq 2$ for all $(x, y) \in A \times A \setminus \{(a, c), (c, a)\}, x \neq y$. Since the McKelvey uncovered set consists of all alternatives and no symmetries exist in T_β, it is not possible to say immediately which alternatives f has to choose. Yet, using two auxiliary tournaments, we will show that $f(T_\beta)$ can be determined exactly.

If an agent β with preferences $c \; P_\beta \; a$ leaves the electorate, the majority graph of $R'_{-\beta}$ equals T, regardless of how b and d are ranked. We know from before that

$$f(T) = {}^1\!/_3\, a + {}^1\!/_3\, c + {}^1\!/_3\, d.$$

Since f satisfies *SD*-participation, it may not be the case that $f(R'_{-\beta}) \; P_\beta^{SD} \; f(R')$, hence, by the flexibility of b and d, we conclude that

$$f(T_\beta)(c) \geq {}^1\!/_3.$$

On the contrary, if another agent β' equipped with preferences $a \; P_{\beta'} \; c$ leaves the electorate, the majority graph of R' changes to a tournament that is isomorphic to T and in which a is covered. Consequently,

$$f(R'_{-\beta'}) = {}^1\!/_3\, b + {}^1\!/_3\, c + {}^1\!/_3\, d.$$

As for β, it must also hold for β' that $f(R'_{-\beta'}) \; P_\beta^{SD} \; f(R')$ is not the case. Employing this, we can deduce two (in)equalities:

$$f(T_\beta)(b) = f(T_\beta)(d)$$

$$f(T_\beta)(a) + f(T_\beta)(c) \leq {}^1\!/_3$$

The equality follows since we can freely arrange b and d in β''s preference relation and the fact that $f(T_\beta)(c) \geq {}^1\!/_3$ by the proceeding agreement. Together with $f(T_\beta)(c) \geq {}^1\!/_3$ we therefore directly get

$$f(T_\beta) = {}^1\!/_3\, b + {}^1\!/_3\, c + {}^1\!/_3\, d.$$

On the other hand, any preference profile $R'' \in \mathcal{R}^{\mathcal{F}(\mathbb{N})}$ inducing majority graph T_γ necessarily results in the lottery

$$f(T_\gamma) = {}^1\!/_4\, a + {}^1\!/_4\, b + {}^1\!/_4\, c + {}^1\!/_4\, d$$

because of neutrality.

Finally note that an agent γ with preferences $R_\gamma \colon \{a, c\}, d, b$ joining a suitable electorate with preference profile R'' changes the majority graph of R'' in

		SD-efficient	*ex post* efficient	unanimous	unrestricted
very strong *SD*-part.	majoritarian	–	–	–	–
	pairwise	–	–	–	?
	anonymous and neutral	?	+	+	+
strong *SD*-part.	majoritarian	–	–	–	++
	pairwise	+	+	+	++
	anonymous and neutral	+	+	+	++
SD-part.	majoritarian	–	–	++	++
	pairwise	++	++	++	++
	anonymous and neutral	++	++	++	++

Table 1: Existence of SDSs combining certain notions of efficiency and participation. + and ++ indicate the existence of SDSs satisfying single-agent participation and group-participation, respectively.

exactly the way, that it equals T_β afterwards. From above, we know that,

$$f(T_\beta) = {}^1\!/_3\, b + {}^1\!/_3\, c + {}^1\!/_3\, d.$$

Since $f(T_\gamma)\ P_\gamma^{SD}\ f(T_\beta)$, agent γ has the possibility of *SD*-manipulation by strategic abstention contradicting the initial assumption that f satisfies *SD*-participation. This concludes the proof. $\square$

6. CONCLUSIONS

We analyzed to which extent efficiency and participation are compatible in probabilistic social choice. Our results are summarized in Table 1 for abstention by single agents and groups of agents. Positive results carry over from group-participation to participation, from stronger to weaker notions of efficiency and participation, and from majoritarianness to pairwiseness to anonymity/neutrality. As for impossibilities, the implications are exactly the other way round.

Briefly summarized, we have seen that the SDS *BOR* (which yields the uniform lottery over all Borda winners) satisfies important desirable properties. Apart from the strongest notion of efficiency examined in this paper, *SD*-efficiency, *BOR* also fares well in terms of resistance against manipulation by strategic abstention both for single agents as well as for groups of agents. This result is of special interest since it separates strong *SD*-participation from the related notion of strong *SD*-strategyproofness, which is incompatible with anonymity, neutrality, and *SD*-efficiency [8].[3]

For *RSD*, we were able to show very strong *SD*-participation, i.e., any agent (who is not already perfectly happy with the outcome) can improve his expected utility by participating. It seems as if this property is satisfied by no SDS other than variations of *RSD* and convex combinations of several SDSs including *RSD*. It remains open whether this observation can lead to a characterization of *RSD* and thus a deeper understanding of very strong *SD*-participation. Interestingly and in contrast to *BOR*, *RSD* does not satisfy *SD-group*-participation.

[3]Interestingly, *BOR* is known to be particularly vulnerable to strategic manipulation as it is *single-winner manipulable* [36], which means that it not only violates strong *SD*-strategyproofness but strategyproofness with respect to *any* lottery extension.

Theorem 8 has established that, when restricting attention to majoritarian SDSs, *ex post* efficiency and *SD*-participation are incompatible. This obviously also holds for the weaker but complete *downward lexicographic* and *upward lexicographic* lottery extension [14] as well as the recently defined *pairwise comparison* lottery extension [4, 5]. It is unknown whether this result still holds when further strengthening the lottery extension applied to, e.g., *bilinear dominance*. Two other important open problems that remain are whether there is an SDS that satisfies *SD*-efficiency and very strong *SD*-participation, and whether there is a pairwise SDS that satisfies very strong *SD*-participation.

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